OCR
Oxford Cambridge and RSA

# Wednesday 13 October 2021 – Afternoon

## AS Level Mathematics B (MEI)

**H630/02** Pure Mathematics and Statistics

**Time allowed: 1 hour 30 minutes**

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**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

**ADVICE**
- Read each question carefully before you start your answer.

# Formulae AS Level Mathematics B (MEI) (H630)

## Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

## Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

## The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of $X$ is $np$

## Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions.

**1** Find the coefficient of $x^4$ in the expansion of $(1+3x)^6$. **[3]**

**2** Mia rolls a six-sided die 24 times and records the scores. She displays her results in a vertical line chart. This is shown in **Fig. 2.1**.

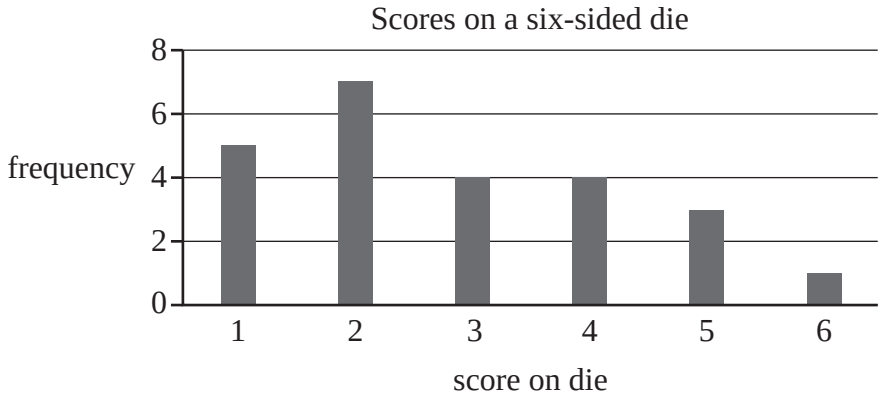


**Fig. 2.1**

**(a)** Describe the shape of the distribution. **[1]**

She repeats the experiment, but this time she rolls the die 50 times. Her results are displayed in **Fig. 2.2**.

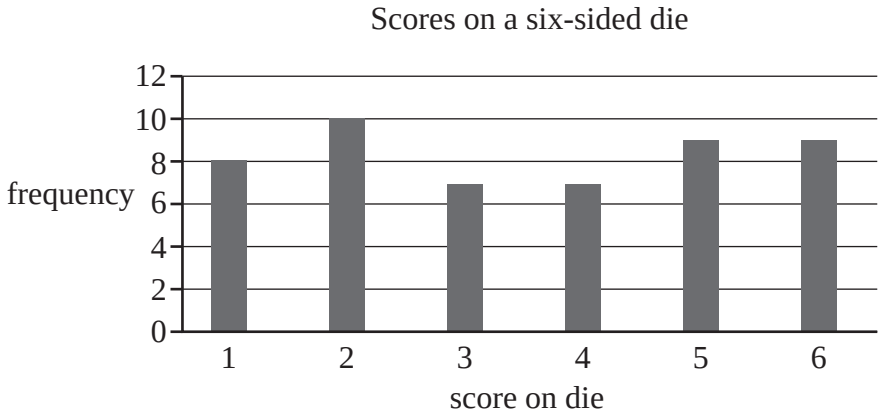


**Fig. 2.2**

Her brother Kai rolls the same die 1000 times and displays his results in a similar diagram.

**(b)** Assuming the die is fair, describe the distribution you would expect to see in Kai's diagram. **[1]**

**3** **In this question you must show detailed reasoning.**

You are given that $\tan 30° = \frac{1}{\sqrt{3}}$.

Explain why $\tan 690° = -\frac{1}{\sqrt{3}}$. **[3]**

**4** Find $\int\left(9x^2 + \frac{6}{\sqrt{x}}\right)\mathrm{d}x$. **[4]**

**5** In 2019 scientists developed a model for comparing the ages of humans and dogs.
According to the model,

$Y = A\ln X + B$

where $X$ = dog age in years and $Y$ = human age in years.

For the model, it is known that when $X = 1$, $Y = 31$ and when $X = 12$, $Y = 71$.

**(a)** Find the value of $B$. **[1]**

**(b)** Determine the value of $A$, correct to the nearest whole number. **[2]**

Use the model, with the exact value of $B$ and the value of $A$ correct to the nearest whole number, to answer parts **(c)** and **(d)**.

**(c)** Find the human age corresponding to a dog age of 20 years. **[1]**

**(d)** Determine the dog age corresponding to a human age of 120 years. **[3]**

**6** The probability distribution for the discrete random variable $X$ is shown below.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $3p^2$ | $0.5p^2 + 2p$ | $1.5p$ | $1.5p^2 + 0.5p$ |

**(a)** Determine the value of $p$. **[4]**

**(b)** Determine the modal value of $X$. **[2]**

**7** The pre-release material contains information about health expenditure. **Fig. 7.1** shows an extract from the data.

| **Country** | **Health expenditure (% of GDP)** |
|---|---|
| Algeria | 7.2 |
| Egypt | 5.6 |
| Libya | 5 |
| Morocco | 5.9 |
| Sudan | 8.4 |
| Tunisia | 7 |
| Western Sahara | #N/A |
| Angola | 3.3 |
| Benin | 4.6 |
| Botswana | 5.4 |
| Burkina Faso | 5 |

**Fig. 7.1**

**(a)** Explain how the data should be cleaned before any analysis takes place. **[1]**

Kareem uses all the available data to conduct an investigation into health expenditure as a percentage of GDP in different countries.

He calculates the mean to be 6.79 and the standard deviation to be 2.78.

**Fig. 7.2** shows the smallest values and the largest values of health expenditure as a percentage of GDP.

| **Smallest values of Health expenditure (% of GDP)** | **Largest values of Health expenditure (% of GDP)** |
|---|---|
| 1.5 | 11.7 |
| 1.9 | 11.9 |
| 2.1 | 13.7 |
| | 13.7 |
| | 16.5 |
| | 17.1 |
| | 17.1 |

**Fig. 7.2**

**(b)** Determine which of these values are outliers. **[4]**

Kareem removes the outliers from the data and finds that there are 187 values left. He decides to collect a sample of size 30.

He uses the following sampling procedure.

Assign each value a number from 1 to 187.

Generate a random number, $n$, between 1 and 13.

Starting with the $n$th value, choose every 6th value after that until 30 values have been chosen.

**(c)** Explain whether Kareem is using simple random sampling. **[2]**

**8** With respect to an origin O, the position vectors of the points A and B are

$\overrightarrow{OA} = \begin{pmatrix} -3 \\ 20 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$.

**(a)** Determine whether $\left|\overrightarrow{AB}\right| > 200$. **[2]**

The point C is such that $\overrightarrow{AC} = \begin{pmatrix} 18 \\ -24 \end{pmatrix}$.

**(b)** Determine whether A, B and C are collinear. **[2]**

**9** Arun, Beth and Charlie are investigating whether there is any association between death rate per 1000 and physician density per 1000. They each collect a random sample of size 10.

Arun's sample is shown in **Fig. 9.1**.

| | death rate per 1000 | physician density per 1000 |
|---|---|---|
| Canberra | 7.2 | 3.62 |
| Dhaka | 5.3 | 0.49 |
| Brasilia | 6.8 | 2.23 |
| Yaounde | 9.3 | 0.08 |
| Zagreb | 12.5 | 3.08 |
| Tehran | 5.4 | 1.16 |
| Rome | 10.7 | 4.14 |
| Tripoli | 3.8 | 2.09 |
| Oslo | 7.9 | 4.51 |
| Abuja | 9.7 | 0.35 |

**Fig. 9.1**

**(a)** Explain whether or not Arun collected his data from the pre-release material, or whether it is not possible to say. **[1]**

**8**

Beth and Charlie collected their samples from the pre-release material. Each of them drew a scatter diagram for their samples. The samples and scatter diagrams are shown in **Figs. 9.2** and **9.3**.

| **Beth's sample** | death rate per 1000 | physician density per 1000 |
|---|---|---|
| Sudan | 6.7 | 0.41 |
| Cambodia | 7.4 | 0.17 |
| Gabon | 6.2 | 0.36 |
| Seychelles | 7 | 0.95 |
| Mexico | 5.4 | 2.25 |
| Kuwait | 2.3 | 2.58 |
| Haiti | 7.5 | 0.23 |
| Maldives | 4 | 1.04 |
| Nauru | 5.9 | 1.24 |
| Jordan | 3.4 | 2.34 |

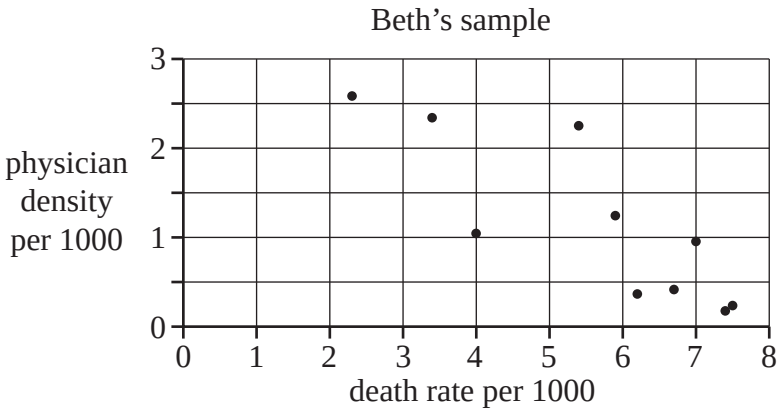


**Fig. 9.2**

| **Charlie's sample** | death rate per 1000 | physician density per 1000 |
|---|---|---|
| Vanuata | 4 | 0.17 |
| Solomon Islands | 3.8 | 0.2 |
| N. Mariana Islands | 4.9 | 0.36 |
| Nauru | 5.9 | 1.24 |
| United Kingdom | 9.4 | 2.81 |
| Portugal | 10.6 | 3.34 |
| North Macedonia | 9.6 | 2.87 |
| Faroe Islands | 8.8 | 2.62 |
| Bulgaria | 14.5 | 3.99 |
| St. Kitts and Nevis | 7.2 | 2.52 |

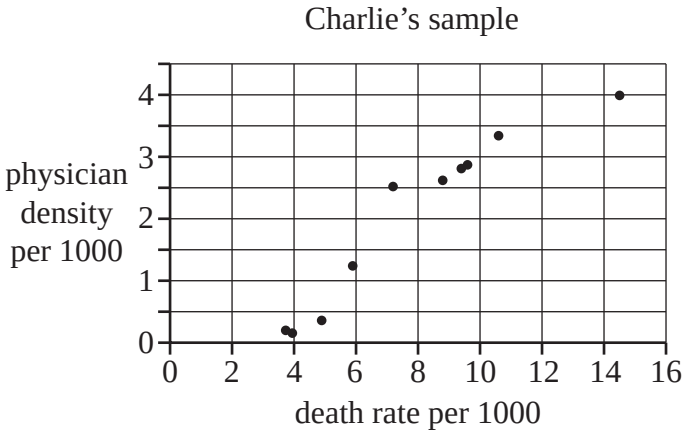


**Fig. 9.3**

Arun states that Charlie's sample and Beth's sample cannot **both** be random for the following reasons.

- Both samples include Nauru – there should not be any common values.
- Beth's diagram suggests a negative association between death rate and physician density, whereas Charlie's diagram suggests a positive association. If both samples are random the same relationship would be suggested.

**(b)**
- Explain whether Arun's reasons are valid.
- State whether or not Arun is correct, or whether it is not possible to say. **[3]**

Kofi collects a sample of 10 African countries and 10 European countries. The scatter diagram for his results is shown in **Fig. 9.4**.

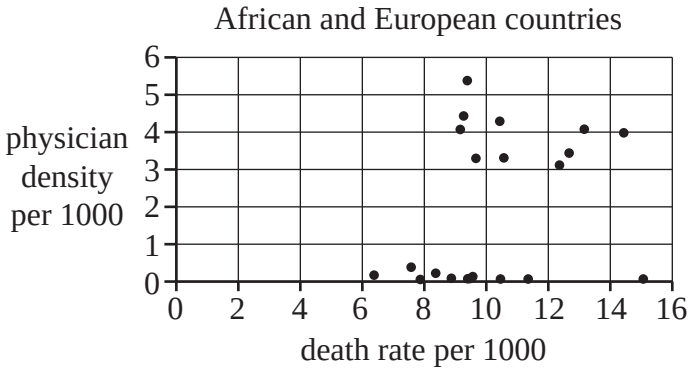


**Fig. 9.4**

**(c)** On the copy of **Fig. 9.4** in the Printed Answer Booklet, use your knowledge of the pre-release material to identify the points representing the 10 European countries, justifying your choice. **[1]**

**10 In this question you must show detailed reasoning.**

The points P, Q and R have coordinates $(-3, 7)$, $(11, 5)$ and $(9, -9)$ respectively.

**(a)** Show that PQ is perpendicular to QR. **[3]**

A circle passes through P, Q and R.

**(b)** Determine the coordinates of the centre of the circle. **[3]**

**11** James is investigating the amount of time retired people spend each day using social media. He collects a sample by advertising in a local newspaper for people to complete an online survey.

**(a)** State

- the name of the sampling technique he is using,
- **one** disadvantage of using this technique. **[2]**

James processes his data in order to draw a histogram. His table of results is shown below.

| Time spent using social media in minutes | 0 – | 15 – | 30 – | 60 – | 120 – 240 |
|---|---|---|---|---|---|
| Number of people per minute | 12.2 | 14.0 | 8.4 | 7.3 | 3.1 |

**(b)** Show that the size of the sample is 1455. **[2]**

**(c)** Calculate an estimate of the probability that a retired person spends more than an hour per day using social media. **[2]**

**12** A manufacturer of steel rods checks the length of each rod in randomly selected batches of 10 rods.

100 batches of 10 rods are checked and $x$, the number of rods in each batch which are too long, is recorded.

Summary statistics are as follows.

$n = 100 \qquad \sum x = 210 \qquad \sum x^2 = 604$

**(a)** Calculate

- the mean number of rods in a batch which are too long,
- the variance of the number of rods in a batch which are too long. **[3]**

Layla decides to use a binomial distribution to model the number of rods which are too long in a batch of 10.

**(b)** Write down the parameters that Layla should use in her model. **[2]**

**(c)** Use Layla's model to determine the expected number of batches out of 100 in which there are exactly 2 rods which are too long. **[3]**

**13 In this question you must show detailed reasoning.**

The equation of a curve is $y = 3x + \frac{7}{x} - \frac{3}{x^2}$.

Determine the coordinates of the points on the curve where the curve is parallel to the line $y = 2x$.

**[9]**

**END OF QUESTION PAPER**